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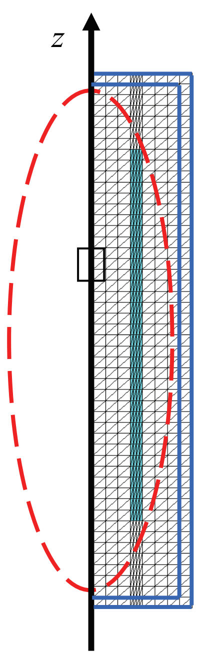
Introduction

Computation of low-frequency field penetration through conducting and magnetic materials is important for accessing interference within devices and facilities. Another use is the prediction of naval vessel vulnerability to detection caused by the field signatures of their internal electrical machinery. A hybrid procedure is used for predicting the penetration of low-frequency fields through a cylindrical structure. The structure is assumed to be rotationally symmetric but otherwise can be inhomogeneous, including layers or rings of differing material. The internal source is an axially oriented multi-turn coil. Field computation is formulated using a finite element solution within the interior region coupled to a spheroidal field expansion in the unbounded exterior. Comparisons are made to field measurements for a coil inside a steel pipe at frequencies of 1Hz and 50Hz.

Formulation

The hybrid method couples a finite element method (FEM) solution within an interior region to a spheroidal field expansion in an overlapping exterior region, as depicted in Fig. 1a. The mesh encloses the coil source and the cylindrical shield.



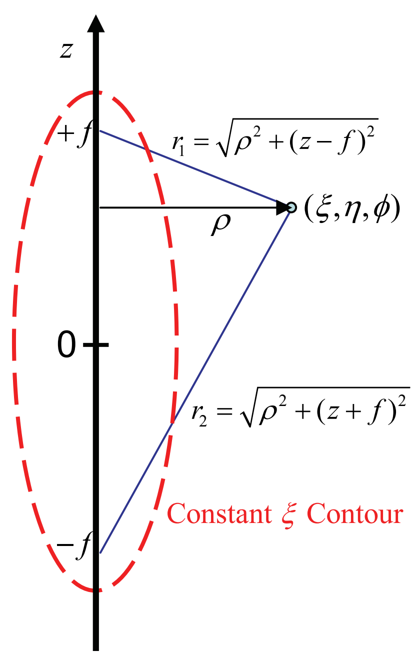


Fig. 1 (a) Finite element mesh with coil and shield; (b) spheroidal coordinates.

Figure 1(b) depicts the spheroidal geometry with foci at $z = \pm f$ and coordinates $\xi = \left(r_2 + r_1\right)/2f$, $\eta = \left(r_2 - r_1\right)/2f$ and ϕ the azimuth angle. A constant ξ contour encloses the coil and the shield. In the free-space region exterior to this contour the quasi-static electric field can be represented by the expansion [1],

$$E_\phi = -j\omega\,\mu_0 \sum_{n=1}^{\infty} c_n\, Q_n^1(\xi)\, P_n^1(\eta) \tag{1}$$

where $P_n^1(\eta)$ and $Q_n^1(\xi)$ are associated Legendre functions of the first and second kinds. Magnetic field components are found using the curl of the electric field,

$$\begin{aligned} H_\phi &= -\sum_{n=1}^{\infty} c_n \left\{ \frac{dQ_n^1(\xi)}{d\xi}\frac{d\xi}{dz} P_n^1(\eta) + Q_n^1(\xi)\frac{dP_n^1(\eta)}{d\eta}\frac{d\eta}{dz} \right\} \\ H_z &= \sum_{n=1}^{\infty} c_n \left\{ \frac{dQ_n^1(\xi)}{d\xi}\frac{d\xi}{d\rho} P_n^1(\eta) + Q_n^1(\xi)\frac{dP_n^1(\eta)}{d\eta}\frac{d\eta}{d\rho} + \frac{1}{\rho} Q_n^1(\xi)\, P_n^1(\eta) \right\} \end{aligned} \tag{2}$$

A 500-turn coil provides quasi-static fields which are computed by superposition of elliptic integral expressions for single-turn fields [2].

Within the finite element region the electric field can be shown to satisfy,

$$\frac{\partial}{\partial\rho}\left[\frac{1}{\mu_r\,\rho}\frac{\partial}{\partial\rho}(\rho E_\phi)\right] + \frac{\partial}{\partial z}\left[\frac{1}{\mu_r}\frac{\partial E_\phi}{\partial z}\right] + \omega^2 \mu_0 \varepsilon_0\, \varepsilon_r\, E_\phi = 0 \tag{3}$$

where relative material properties $\varepsilon_r(\rho, z)$ and $\mu_r(\rho, z)$ may be inhomogeneous and complex (to include losses) [3]. FEM solution for $E_\phi(\rho, z)$ at the inner mesh nodes is obtained for: (1) each expansion mode in (1) applied as a boundary condition on the outer mesh contour in Fig. 1(a) with no coil field, and; (2) coil excited by a specified current with zero E_ϕ on the outer mesh boundary. For case (2) inside the mesh, the computed coil field is subtracted from $E_\phi(\rho, z)$ at nodes enclosing the coil, thus forcing a partial solution (sans the coil field) at nodes which overlap the coil.

Expansion coefficients in (1) and (2) are found using the unimoment method [3]. Note that the total E_ϕ inside the mesh can be assembled using a $\{c_n\}$-weighted superposition of the FEM solutions found due to the expansion mode boundary conditions, plus that due to the coil field. This superposition of FEM solutions for E_ϕ is set equal to the exterior expansion in (1) at the nodes of the interior mesh contour which just encloses the spheroid in Fig. 1(a). This equality forms an overdetermined system which is solved for the set of $\{c_n\}$ using a pseudo-inverse. The $\{c_n\}$ set is used to assemble the mesh fields and expand (1) and (2).

Experimental Validation

Magnetic fields were measured for an axially directed 500-turn coil placed at the midpoint of a $3.048m$ long steel pipe having outer diameter of $0.324m$ and thickness of $9.78mm$, with parameters $\varepsilon_r = 1$, $\mu_r = 176$ and $\sigma = 4.7 \times 10^6 S/m$. As shown in Fig. 2, a tri-axial magnetic sensor was placed at a radial distance of $0.950m$ from the pipe centerline. Field measurements were made at 43 equispaced axial positions of the sensor, with displacements from the pipe and coil center ranging from $z = -2.1336m$ to $+2.1336m$.

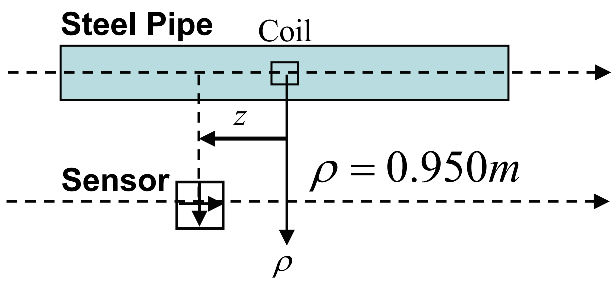


Fig. 2 Measurement geometry depicting steel pipe with centered coil and axially displaced tri-axial magnetic field sensor.

Measured radial and axial B-field components due to peak coil current $I = 1A$ at 1Hz and 50Hz are compared to computations in Figs. 3 and 4.

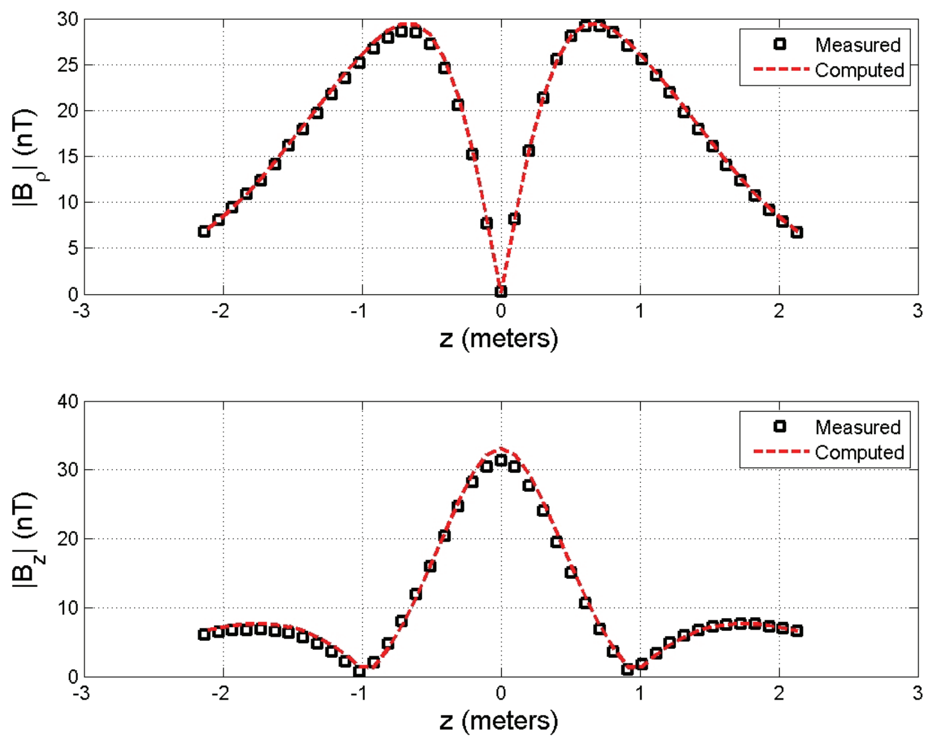


Fig. 3 Measured and computed B-field components at $\rho = 0.95m$ for 1Hz

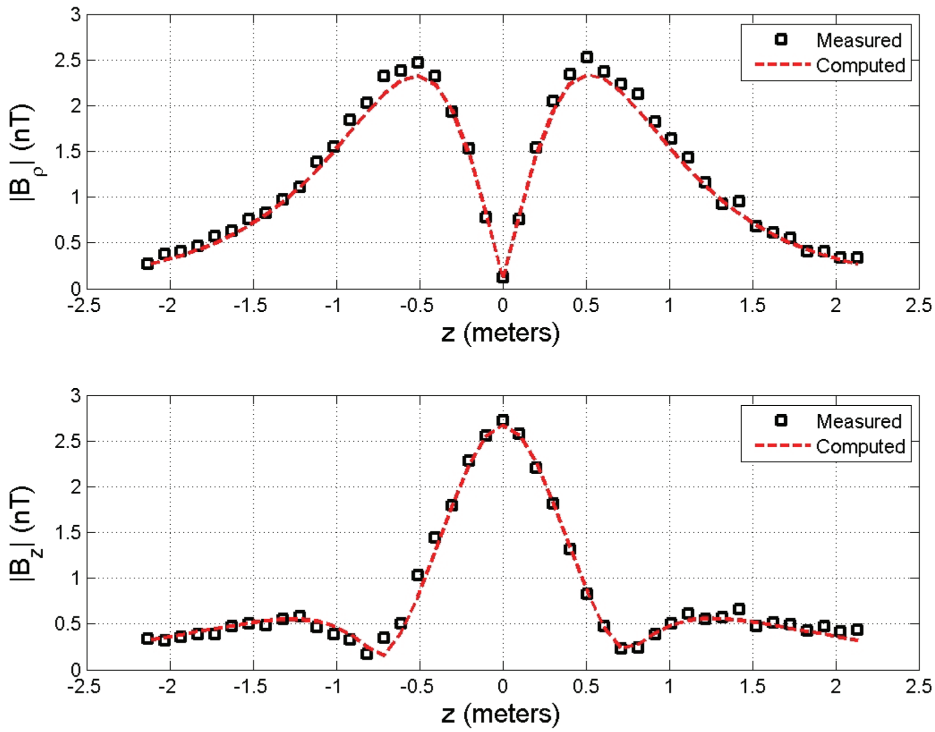


Fig. 4 Measured and computed B-field components at $\rho = 0.95m$ for 50Hz

Conclusion

A hybrid method combining finite elements and a spheroidal harmonic expansion is formulated and then validated. Excellent agreement is shown for measurements of penetrating fields produced by a 500 turn coil inside a steel pipe at frequencies of 1 Hz and 50 Hz. This method can be applied to axisymmetric structures composed of arbitrary materials. It will be employed to design shielding structures composed of several conducting and magnetic materials, optimized for a specified frequency range under constraints of shield thickness and weight.

Acknowledgements

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References

[1] I. R. Capoglu and G. S. Smith, "The input admittance of a prolate-spheroidal monopole antenna fed by a magnetic frill," *IEEE Trans. Antennas Propag.*, vol. AP-54, pp. 572-585, Feb. 2006.

[2] W. R. Smythe, *Static and Dynamic Electricity*, Third Edition, New York: Hemisphere, 1989.

[3] M. A. Morgan and K. K. Mei, "Finite-element computation of scattering by inhomogeneous penetrable bodies of revolution," *IEEE Trans. Antennas Propag.*, vol. AP-27, pp. 202-214, Mar. 1979.